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ORIGINAL ARTICLE

Prediction of burr formation during face milling using a hybrid GMDH network model with optimized cutting conditions

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Abstract In this paper, a combined hybrid group method for data handling and optimization approach is introduced to predict burr types formed during face milling. The hybrid group method for data handling (hybrid GMDH) network was constructed for realizing predictive models for the machining of aluminum alloy, and differential evolution was selected for the optimization of burr formation problem resulting in finding optimal parameter for minimizing burr formation. Burr type was included as a parameter resulting in a classification scheme in which the burr type becomes the group label and it is therefore possible in the future to classify a machining process into any of these burr types. The resulting hybrid GMDH output was in agreement with experimental results, thereby validating the proposed scheme for modeling and prediction of burr formation in milling operations.

Keywords Face milling · Burr · Cutting parameters · Optimization · Hybrid GMDH network

Abbreviations

ANN	Artificial neural network
ANOVA	Analysis of variance
DE	Differential evolution
eGMDH	Enhanced GMDH
GMDH	Group method for data handling
SOMA	Self-organizing migrating algorithm

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1 Introduction

Milling is the most versatile of the conventional machine tools. In concept, milling is very straightforward. A cutter is held in a chuck, which rotates at a controlled speed. The cutter is suspended over a work surface whose location can be precisely controlled. The part to be machined is securely fastened to the work surface, and the work surface is moved underneath the cutter. Appropriate choices of cutter type, depth of cut, and speed determine the final shape. Face milling operation is a process of removing material by feeding the workpiece past a rotating multipoint cutter. It is a high metal removal rate process, which is more suitable for use in mass production. Therefore, face milling processes are widely used in the manufacturing industry.

Materials best suited for milling are the softer metals and plastics. Aluminum and brass are two commonly milled metals; Teflon and Delrin are commonly milled plastics. However, the ability to mill a metal is typically limited only by the hardness of the cutter. Special cutters can be obtained for milling harder materials and refractory. Alternatively, very sharp cutters are available for plastics and even wood. Milling can be performed under computer control. Such mills called computer numerical control (CNC) mills are becoming increasingly common in small machine shops. There are numerous variations on these mills; the most interesting are CNC mills that machine simple circuit boards. The advantages of milling include the fact that it is very good for one-off objects, virtually any material can be milled with a proper cutter, complex parts with high detail can be milled, and tolerances of 0.0254 to 0.0762 mm are possible.

Burrs are defined as undesirable projections of materials beyond the edge of the workpiece arising because of plastic deformation during machining [1]. Burrs formed during face milling operations are difficult to characterize because there are several parameters with complex interactions that affect the cutting process. Burrs can cause many problems during inspection, assembly, and automated manufacturing of precision components. Therefore, it is desirable for precision parts to be burr free.

In general, there are two ways to deal with troublesome burrs. One option is deburring. As mentioned by Gillespie [2], burr removal processes are often costly, accounting for up to 30% of the total cost of precision machining parts. Also, deburring processes are hard to generalize because they vary according to manufacturing circumstances. The other choice is burr planning with parameter optimization [3–6], which not only prevents the formation of burrs or minimizes their negative influences, but also predicts the types and locations of burrs by designing and analyzing manufacturing processes and parameters. For this approach, a careful investigation of process parameters and their interactions is necessary.

When conducting burr research in areas such as burr planning and burr removal, relevant burr formation mechanism(s) need to be understood. Since theoretical approaches are usually not available, researchers have concentrated on experimental studies to identify the effects of machining parameters on burr formation [7–12].

Among these, Chern [9] investigated exit burrs during face milling on aluminum. He observed four different types of burr with variations in depth of cut and in-plane exit angle: knife-edge, curl, wave, and secondary. The first three types of burr are primary burrs that have to be removed. The secondary burrs are relatively small burrs that remain after the main portions of the larger primary burrs are cut off. They typically do not pose a problem and therefore do not require deburring.

Several practical applications have demonstrated the need for an optimized set of specific machining parameters. Experimental studies have shown that burrs can be minimized or controlled when adequate machining parameters are selected; however, the results of these studies tend to be limited to certain process parameters, such as range and materials, owing to the complicated interactions among parameters. Recently, the Taguchi method, a widely used systematic optimization application in the design and analysis of experiments, has been successfully introduced in various manufacturing areas including burr formation [3, 13]. Apart from parameter optimization, some researchers have focused on online prediction and classification methods for burrs generated during the manufacturing process.

Artificial neural network (ANN) remains the most popular approach for prediction of burr sizes in machining. Tseng and Chiou [14] predicted burr height using ANN, using the Taguchi method for training selected input and output samples. Karnik et al. [15] employed the Taguchi method for optimization of simultaneous minimization of burr height and burr thickness in drilling operations. Lee and Dornfeld [16] performed cutting parameter optimization with respect to burr minimization in face milling, and a subsequent burr-type prediction scheme based on the optimal results was proposed. The Taguchi method was used for the experimental parameter optimization for minimum burr heights. They examined the performance characteristics in more detail by employing analysis of variance (ANOVA) and used the optimized results to normalize the input vectors of an ANN. Finally, they constructed an ANN for the burr-type prediction, giving reasons that burr types are more effective than simple dimensions including burr heights for the evaluation of edge finishing quality and suitability of deburring as supported in [17, 18]. The burr-type prediction approach proposed by Lee and Dornfeld [16] is interesting because it facilitated the construction of an ANN burr-type prediction classifier in which burr heights less than the threshold value of 0.45 mm were treated as type 1 and those greater than 0.45 mm were treated as type 2. In other words, type 1 and type 2 are the labels for the milling burr formation.

More recently, the group method for data handling (GMDH) network, which was initially employed by researchers for modeling complex systems, has been employed in solving manufacturing problems, for example, as in tool wear investigation [19]. The GMDH networks which are quite competitive are known to be inductive while ANN networks are known to be deductive and require users to carry out initial extensive experimentation in determining the number of nodes in the different layers.

In the current study, a novel approach based on hybrid GMDH [20, 21] is proposed for prediction of burr size as well as for classification of burr types. The differential evolution (DE) approach which is an integral part of the hybrid GMDH system employed is used to determine the optimal cutting conditions and burr size for minimizing burr formation. The results of the experiments carried out in this paper shows that the proposed hybrid GMDH-based approach is more robust (flexible, easier to use, and accurate) than the ANN approach that is popularly utilized so far in the literature for the burr prediction problem.

2 Modeling the mill burr problem using hybrid de-group method of data handling

The hybrid DE-GMDH algorithm that is utilized for the mill burr problem reported in this paper consists of two components: (a) the DE structural optimization module, and (b) the GMDH parametric optimization module.

2.1 Group method of data handling

The algorithm of GMDH [22] was first introduced by Ivakhnenko in 1966 [22]. Its main purpose was the identification of relations in large complex nonlinear multidimensional systems, their approximation and prediction. GMDH searches for optimal structure within the space of multipolynomial functions $g: \boldsymbol{R}^n \rightarrow \boldsymbol{R}$ which it realizes as a multilayered polynomial network. The main idea behind the algorithm is obtaining a mathematical model of the analyzed object produced in an automated heuristic driven self-organizing learning process.

Ivakhnenko employed the Kolmorovo–Gaborov sentence [22], which proves that every function $y_n = f(X)$ can be represented by an infinite Volterra–Kolmogorov–Gabor (VKG) polynomial [22] of the form:

$$y_n = a_0 + \sum_{i=1}^{M} a_i x_i + \sum_{i=1}^{M} \sum_{j=1}^{M} a_{ij} x_i x_j + \sum_{i=1}^{M} \sum_{j=1}^{M} \sum_{k=1}^{M} a_{ijk} x_i x_j x_k \ldots \quad (1)$$

where $X = (x_1, x_2, \ldots, x_M)$ is the vector of input variables and $A = (a_1, a_2, \ldots, a_M)$ is the vector of coefficients or weights. We note that $A = \{a_i, a_{ij}, a_{ijk} \ldots\}$ is the vector of summand coefficients. This formula exhausts all combinations among input vectors (units), so it is considered to be the complete polynomial description of a system model. However, to determine the coefficients of the polynomial (Eq. 1) for general nonlinear system is rather difficult because they depend on not only the number of polynomial terms used but also the number of variables and data. This is the discrete time analogue of a continuous time Volterra series and can be used to approximate any stationary random sequence of physical measurements.

In the GMDH algorithm, the Volterra–Kolmogorov–Gabor (VKG) series is estimated by a cascade of second order polynomials using only pairs of variables [22, 23]. The corresponding network can be constructed from simple polynomial and delay elements. The main function of the model is based on forward propagation of signal through nodes of the GMDH net similar to principle used, e.g., in classical neural nets—input signal is applied to input nodes, the outputs of which are then distributed through the structure to upper layers where appropriate mathematical combinations are carried out. Each layer consists of simple nodes each performing its own polynomial transfer function and passing its output to nodes in the next layer. The output of the last layer (consisting of only one node) is the output of the whole net.

The coefficients of nodes' transfer functions are estimated during the learning phase within, of which the whole structure is being automatically built up. This inductive approach to determining the model structure notably reduces the amount of a priori knowledge required from the user and allows selecting a structure that follows best the given dataset. During the evolution of the learning procedure, network branches that do not contribute significantly to the specific output can be pruned, thereby allowing only the dominant causal relationship to evolve. An enhanced GMDH (eGMDH) which performs better than the traditional GMDH is described in [19].

The GMDH network model is constructed during the learning process by the following five procedures:

Step 1: *Separating the original data into the training and test sets.* The original dataset is separated into the training and test sets. Training data are used for the estimation of the partial descriptions which describe the partial characteristics of the nonlinear system and the test data are used for organizing the complete description which describes the complete characteristic of the nonlinear system.

Step 2: *Generating combinations of the node input variables in each layer.* All combinations of r input variables are generated before learning each layer. The number of combinations is ${}^{m}C_r = \frac{m!}{r!(m-r)!}$ where m is the number of input variables and r is the number of inputs for each node (usually set to two according to basic model introduced in [22]).

Step 3: *Calculating the optimum partial descriptions.* For each combination, the optimum partial descriptions are calculated, e.g., by applying the regression analysis to the training data (other approaches utilize, e.g., quasi-Newton gradient method etc.). The output variables y_k in the partial descriptions are called as intermediate variables.

Step 4: *Selecting the intermediate variables.* The $\Re$ intermediate variables which give the $\Re$ smallest test errors calculated for *the test dataset* are selected from the generated intermediate variables y_k. Selected $\Re$ intermediate variables are used in following iteration as input variables of the next layer and calculations from procedure 2 to 4 are repeated.

Step 5: *Considering stopping the multilayered iterative computation.* When the error of the test data predictions resulting from the last layer stops decreasing, the iterative computation is terminated. The complete description of the characteristics of the nonlinear system can be constructed by using the optimum partial descriptions generated in each layer in the form of a network.

2.2 Differential evolution scheme

The DE algorithm introduced by Storn and Price [24] is a novel parallel direct search method, which utilizes N_{p} parameter vectors as a population for each generation G. It was primarily designed for continuous domain space formulation. The steps involved in the classical DE are summarized here

Step 1: Initialization
Step 2: Mutation
Step 3: Crossover
Step 4: Selection
Step 5: Stopping criteria

2.2.1 Permutative-based DE

Permutative-based DE differs from continuous DE due to the fact that it can handle permutative-based type combinatorial optimization problems. The mechanisms to cater for this are mainly the way in which initialization is done together with two other schemes for transformation from permutation form into continuous form in step 2 in Section 2.2 and transformation from continuous form into permutation form after step 2 in Section 2.2 [25].

2.2.2 Enhanced permutative-based DE

As the name implies, the enhanced permutative-based DE uses the same basis as the permutative-based DE except that it has more enhancement strategies [26].

2.3 The hybrid DE-GMDH scheme

Classical GMDH has its strength in regression problems. The hybrid DE-GMDH scheme introduced by Onwubolu [20, 21] (see Fig. 1) overcomes the shortcomings of the conventional GMDH algorithm for complex real-life problems. It comprises of several components including structural and parametric optimization schemes.

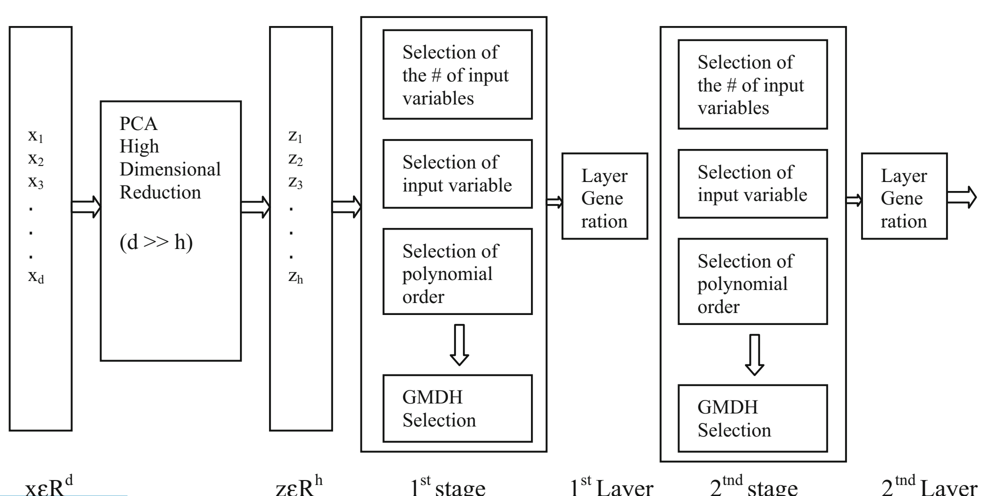
2.3.1 Structural optimization with DE

The DE design is responsible for selecting the number of input variables (attributes), selection of the input variables (attributes), and selection of polynomial order (linear, quadratic, trilinear, tri-quadratic, etc.). From iteration (generation) to iteration (generation) of the DE, the best nodes (neurons) are automatically found based on regression and parametric optimization, and progress is made until termination conditions are satisfied.

The summary of the overall architecture (Fig. 1) of the hybrid DE-GMDH modeling system is as follows:

Step 1: Initialize a population of discrete trial solutions (Section 2.2).
Step 2: Evaluate the objective function (fitness) for each discrete current solutions in the population (Section 2.1).
Step 3: Convert the permutative-based current solutions into continuous current solutions (Section 2.2.2).
Step 4: Apply DE strategy to transform current solutions into new solutions using the inbuilt crossover and mutation schemes (Sections 2.2.1 and 2.2.2).
Step 5: Convert the continuous new solutions into permutative-based new solutions (Section 2.2.2).
Step 6: Repair solutions to realize discrete new solutions of unique values (Section 2.2.2).
Step 7: Improve solutions through standard crossover and mutation schemes (Sections 2.2.1 and 2.2.2).
Step 8: Execute steps 1–6 until reaching a specified cut-off limit on the total number of iterations.

Fig. 1 Overall architecture of the hybrid DE-GMDH modeling system



Step 9: Improve solutions further through local search routine (Sections 2.2.1 and 2.2.2).

To understand network realization, let us take an example. Suppose we have input dataset of five dimensional vectors (record). Each vector has only and only one output. Take a scenario where the order of the polynomial is 2 and the number of inputs to a node is 2. Then, the inputs to a node (neuron) are shown in Table 1, which are just for layer 1. The DE mechanics detailed in Section 2.2 are vehicles for propagating the network from layer to layer.

2.3.2 Parametric optimization with GMDH

The objective function (performance index) is a basic instrument guiding the evolutionary search in the solution space. For the third solution vector (see Table 1), the generated polynomial would be:

$$f(x_2, x_1) = c_1 + c_2x_2 + c_3x_1 + c_4x_2x_1 + c_5x_2^2 + c_6x_1^2$$

where c_1, c_2,…, c_6 are the constants evaluated using training dataset. As discussed in Section 2, the least square technique from multiple regression analysis provides the formula to obtain the coefficients in the following form: $\mathbf{c}=(X^TX)^{-1}X^TY$.

2.4 Procedure of the general learning DE-GMDH algorithm

The general learning procedure for constructing the DE-GMDH model can be described as follows:

1. Create an initial population randomly (DE structures and its corresponding learning parameters).
2. Structural optimization is achieved by the DE variation operators described in Section 2.3.1.
3. If better structure is found, then go to step 4; otherwise go to step 2.
4. Parametric optimization is found using pseudo-inverse or singular value decomposition (SVD) described in Section 2.3.2.
5. If the maximum number of local is reached or no better parameter vector is found for a significantly long time, then go to step 6; otherwise go to step 4.
6. If a satisfactory solution is found, then the algorithm is stopped; otherwise go to step 2.

Table 1 Inputs to a node (neuron)

Vectors	Solution vectors	Node (neuron) inputs
Vector 1	5, 4, 3, 2, 1	5, 4
Vector 2	1, 3, 5, 4, 2	1, 3
Vector 3	2, 1, 4, 5, 3	2, 1
Vector 4	4, 3, 1, 5, 2	4, 3
Vector 5	1, 5, 2, 3, 4	1, 5

Table 2 Some mechanical properties of the aluminum alloy used

Tensile strength (MPa)	180
Yield stress (MPa)	135
BHN	50
constituents	1.2% Mn–1.0% Mg

3 Experiments

3.1 Materials, machine tool, and measurement

The experiment was carried out using an Ajax, model MS knee type, milling machine with a 38-mm-diameter, 19-mm-thick Co-high speed steel (HSS) Kobelco Hi Cut mill cutter having a number of teeth, z=14 to machine aluminum alloy, which is an easy-to-machine material frequently used in burr research. The workpiece dimensions for the experiment are approx. 25 mm×25 mm×50 mm and 32 pieces were used. The Ajax machine allows discrete variation in the spindle speed (48–1,500 rev/min in 12 steps) and the table feed rate (11–500 rev/min in 12 steps). Table 2 shows some mechanical properties of the aluminum alloy used. The diameter of the face milling cutter which is 38 mm is used for cutting the aluminum. The instrument used for measuring the burr height and burr thickness is the Mitutoyo® tool makers' microscope with 0.005 mm resolution.

To measure the burr height, the workpiece was placed sideways and the x-axis of the measuring instrument was set at zero when the cross-wire coincided with the base of the workpiece. The cross-wire was then moved until it coincided with either the maximum and minimum peaks of the burr heights. The maximum and minimum values were also noted and the average of these values was taken that represents burr height at ten different locations.

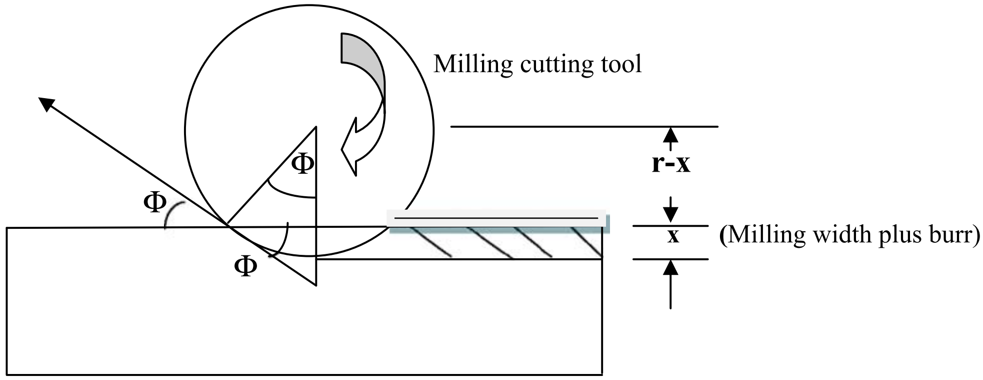
3.2 Selection of cutting parameters

Based on Chern's work [9] in which he derived an equation for the incremental work done for the burr formation in orthogonal cutting, Lee and Dornford [16] concluded that since the negative deformation angle is dependent on the shear angle and the in-plane exit angle [9], and the shear

Table 3 Parameter settings for the experiment

Parameter	Level 1	Level 2	Level 3	Level 4
In-plane exit angle (deg)	19	38	47	55
Depth of cut (mm)	0.5	1.0	1.5	2.0
Feed rate (rev/min)	65	127	264	500
Cutting speed (rpm)	320	410	600	865

Fig. 2 In-plane exit angle in face milling burr formation



angle is determined by the rake angle and material properties [27], it can be stated that

$$\text{Burr formation} \approx (\text{materials function}(v, d_{oc}, \Phi, \text{feed})) \qquad (2)$$

where v is the cutting speed, d_{oc} is the depth of cut, and Φ is the in-plane exit angle. As mentioned above, the in-plane exit angle is a significant factor for determining burr sizes. Therefore, for the multipoint cutting (face milling), four cutting parameters (Table 3), which are critically influential to burr generation and size, were selected. Moreover, these parameters have been widely used in previous research [8–12]. As shown in Fig. 2, the in-plane exit angle during face milling is defined as the angle between the cutting velocity and the exit velocity of the cutter at the end of the workpiece. From Fig. 2, there are a number of observations that could be made:

- The in-plane exit angle Φ is a function of the radial tool engagement, x

$$Cos\ \Phi = \frac{r-x}{r} = \left(1 - \frac{x}{r}\right) \qquad (3)$$

- If x=0, Φ=0
- As x increases, Φ increases
- $0 \leq x \leq 2r \Rightarrow 0 \leq \Phi 0 \leq 180°$

The simple analysis of the in-plane exit angle shown here was used to determine the values of angles used in the experiment reported in this paper. The values of x used in Table 2 are 1, 4, 8, and 6 mm for rows 1–8, 9–16, 17–24, and 25–32, respectively. The tool radius is 19 mm. Hence, the values for these data using Eq. 3 are 19°, 38°, 55°, and 47°, respectively. For example, when x = half of the tool diameter, Φ=60° and so on. Since the in-plane exit angle is known to be a major factor that affects burr formation in milling operation, it becomes necessary to accurately choose values of Φ for experimentation. Figure 3 shows typical burrs generated during the experiments carried out, and these are fairly easy to measure using tool makers' microscope. The design of experiment was based on Table 3.

4 Hybrid GMDH modeling of milling burr process

Three categories of investigations were carried out for the milling burr process: (1) burr height prediction without considering burr type; (2) burr height considering burr type; and (3) optimization of cutting conditions and burr height. The test results for all these three categories are now presented in the subsequent subsections.

Half of the input–output patterns obtained from experimentation using the dataset of Table 4 were used to train the hybrid GMDH. The input patterns were the control parameters, which include the in-plane exit angle, depth of cut, feed, and spindle speed.

The remaining of the input–output patterns obtained from experimentation using the dataset of Table 4 was used to test the hybrid GMDH. The predicted values of burr height are then compared with the respective experimental values based on the external criterion of mean square error, MSE.

4.1 Case 1: Burr height analysis without considering burr types

The milling experiments were carried out in the laboratory using the information contained in Table 4. For analysis of the burr height without consideration of burr types, Table 4 was

Fig. 3 Typical burrs generated during the experiments

submitted to the hybrid GMDH [20] using the principles already discussed regarding training and testing, respectively. The last column which is the burr height B_h^* (mm) was considered as the single output, while the second (in-plane exit angle), third (depth of cut), fourth (feed), and fifth (spindle speed) columns were considered as the inputs.

Figure 4 shows the percentage differences (percent) between the actual and estimated responses for the burr formation problem. The output sequence is {1 3 2 4} but only the first two entries {1, 3} are needed for connection to the preceding node. The coefficients of the final output node are given as follows: {0.0774341, −0.247064, −0.247064, 0.430085, 0.430085, −6.19e−09}. The training and testing errors are PI=1.16e−05 and EPI=1.35e−05, respectively. As could be observed, these error values are extremely small (virtually zero) due to the excellent learning and generalization capabilities of the hybrid GMDH [20] used for studying this milling burr application as could also be confirmed from Fig. 3. The coefficient of correlation, r^2=0.99976. Defining x_1=in-plane exit angle (degree), $x_2=d_t$ (mm), $x_3=f$ (rev/min) and $x_4=N$ (rpm), the model for the burr height is given as:

$$B_h = -11.3217 + 0.6572x_1\ 2.4616x_2 - 0.0027x_3 + 0.0044x_4 \\ -0.044x_1x_2 + 0x_1x_3 - 0x_1x_4 + 0.0133x_2x_3 - 0.0011x_2x_4 \\ -0x_3x_4 - 0.0084x_1^2 - 2.4626x_2^2 - 0x_3^2 - 0x_4^2 \tag{4}$$

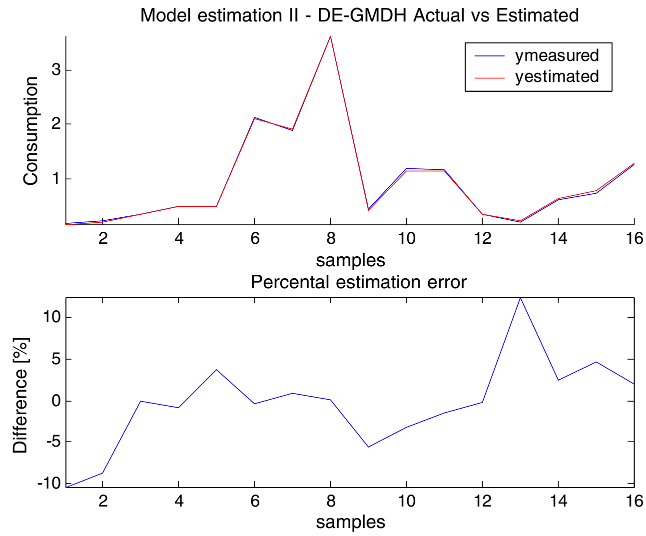


Fig. 4 Percentage differences (percent) between the actual and estimated responses for the burr formation problem (without considering burr type)

Table 4 Experimental input variables and output without considering burr types

Test #	In-plane exit angle (deg)	d_t (mm)	f (rev/min)	Speed (rpm)	B_h^* (mm)
1	19	0.5	65	320	0.162
2	19	0.5	65	410	0.178
3	19	0.5	127	600	0.116
4	19	0.5	127	865	0.223
5	19	1.0	264	320	0.409
6	19	1.0	264	410	0.329
7	19	1.0	500	600	0.176
8	19	1.0	500	865	0.492
9	38	1.5	65	320	0.119
10	38	1.5	65	410	0.477
11	38	1.5	127	600	2.254
12	38	1.5	127	865	2.119
13	38	2.0	264	320	1.399
14	38	2.0	264	410	1.893
15	38	2.0	500	600	3.678
16	38	2.0	500	865	3.629
17	55	0.5	65	320	0.815
18	55	0.5	65	410	0.435
19	55	0.5	127	600	0.335
20	55	0.5	127	865	1.178
21	55	1.0	264	320	1.046
22	55	1.0	264	410	1.164
23	55	1.0	500	600	0.633
24	55	1.0	500	865	0.345
25	47	1.5	65	320	0.274
26	47	1.5	65	410	0.192
27	47	1.5	127	600	0.603
28	47	1.5	127	865	0.605
29	47	2.0	264	320	0.855
30	47	2.0	264	410	0.737
31	47	2.0	500	600	4.946
32	47	2.0	500	865	1.251

4.2 Case 2: Burr height analysis considering burr types

Categorizing burr types used in the work reported in this paper follows the methodology of [16]. Treating burr patterns formed as being similar to surface roughness patterns as shown in Fig. 5, the burr type can be obtained from making reference to the mean burr given as $\overline{h} = \frac{1}{n}\sum_{i=1}^{n} h_i$.

Consequently, burr types can be obtained by considering the difference between the burr mean and height at any point as follows:

$$\text{Burr type} = \begin{Bmatrix} 1 \text{ if } \left(\mathrm{h} - \overline{\mathrm{h}}\right) > 0 \\ 0 \text{ if } \left(\mathrm{h} - \overline{\mathrm{h}}\right) < 0 \end{Bmatrix} \tag{5}$$

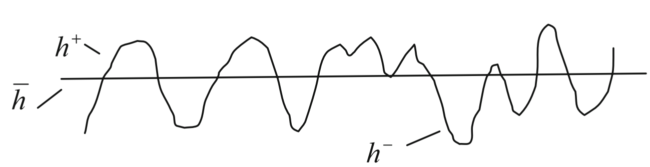


Fig. 5 Burr mean as well as low and high burrs

From the current experimental results, the average burr value is 1.03334375 mm. Hence, burr level less than 1.03334375 mm is considered burr type 1=0 (secondary); and when burr level is above 1.03334375 mm it is considered burr type 2=1 (primary). For analysis of the burr height in which burr types are considered, Table 5 was submitted to the hybrid GMDH [20] using the principles already discussed regarding training and testing, respectively. The last column which is the burr height B_h^*(mm) was considered as the single output, while the second (in-plane exit angle), third (depth of cut), fourth (feed), fifth (spindle speed), and sixth (burr type) columns were considered as the inputs.

Figure 6 shows the percentage differences (percent) between the actual and estimated responses for the burr formation problem. The output sequence is {5 2 3 1 4} but only the first two entries {5, 2} are needed for connection to the preceding node. The coefficients of the final output node are given as follows: {0.108725, −0.680639, 0.271012, 2.03348, −0.680639, −0.000239404}. The training and testing errors are PI=1.14e−05 and EPI=1.33e−05, respectively. As could be observed, these error values are extremely small (virtually zero) due to the excellent learning and generalization capabilities of the hybrid GMDH [20] used for studying this drilling burr application as could also be confirmed from Fig. 6. The coefficient of correlation, r^2=0.999763. Additionally defining x_1 = Burr type, the model for the burr height is given as:

$$
\begin{aligned}
B_h = {} & -4.4234 + 0.0524x_1 + 8.0736x_2 - 0.0298x_3 + 0.0083x_4 + 2.8805x_5 \\
& - 0.0277x_1x_2 + 0.0000x_1x_3 - 0.0000x_1x_4 - 0.0404x_1x_5 \\
& + 0.0227x_2x_3 - 0.0018x_2x_4 - 0.3849x_2x_5 - 0.0000x_3x_4 - 0.0080x_3x_5 \\
& - 0.0018x_4x_5 \\
& - 0.0001x_1^2 - 3.9974x_2^2 + 0.0000x_3^2 - 0.0000x_4^2 + 2.8805x_5^2
\end{aligned}
\tag{6}
$$

Table 5 Experimental input variables and output considering burr types

Test #	In-plane exit angle (deg)	d_t (mm)	f (rev/min)	Speed (rpm)	Burr type	B_h^* (mm)
1	19	0.5	65	320	0	0.162
2	19	0.5	65	410	0	0.178
3	19	0.5	127	600	0	0.116
4	19	0.5	127	865	0	0.223
5	19	1	264	320	0	0.409
6	19	1	264	410	0	0.329
7	19	1	500	600	0	0.176
8	19	1	500	865	0	0.492
9	38	1.5	65	320	0	0.119
10	38	1.5	65	410	0	0.477
11	38	1.5	127	600	1	2.254
12	38	1.5	127	865	1	2.119
13	38	2	264	320	1	1.399
14	38	2	264	410	1	1.893
15	38	2	500	600	1	3.678
16	38	2	500	865	1	3.629
17	55	0.5	65	320	0	0.815
18	55	0.5	65	410	0	0.435
19	55	0.5	127	600	0	0.335
20	55	0.5	127	865	1	1.178
21	55	1	264	320	1	1.046
22	55	1	264	410	1	1.164
23	55	1	500	600	0	0.633
24	55	1	500	865	0	0.345
25	47	1.5	65	320	0	0.274
26	47	1.5	65	410	0	0.192
27	47	1.5	127	600	0	0.603
28	47	1.5	127	865	0	0.605
29	47	2	264	320	0	0.855
30	47	2	264	410	0	0.737
31	47	2	500	600	1	4.946
32	47	2	500	865	1	1.251

4.3 Comparison between case 1 and case 2

In order to ascertain the importance of including the burr type in the prediction models, the results of case 1 and case 2 are considered as tabulated in Table 6. As could be observed, when the burr types are included in the modeling process, the training and testing errors are much less than when burr types are not considered. Consequently, it could be concluded that including burr types in modeling burr formation results in both better learning and generalization capabilities of the hybrid GMDH network. This conclusion

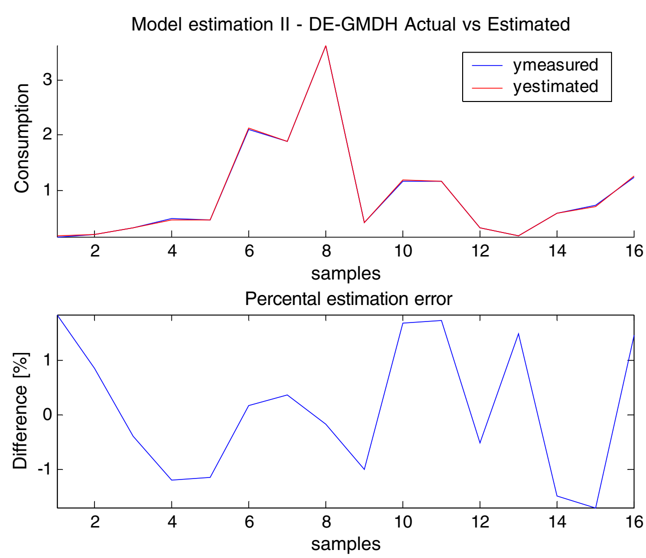


Fig. 6 Percentage differences (percent) between the actual and estimated responses for the burr formation problem (considering burr type)

Table 6 Training and testing errors for case 1 and case 2

	Training error, PI	Testing error, EPI
Case 1: without burr type	0.0000116	0.0000135
Case 2: with burr type	0.00000517	0.000007

agrees with similar investigation carried out using the ANN approach [16].

5 Optimization of cutting parameters for burr prediction

The hybrid GMDH burr size model developed is utilized by the continuous DE for optimization in order to determine the optimal combinations of feed (*f*), drill diameter (*d*), and point angle (θ) that result in minimizing burr height and burr thickness. The optimization problem is essentially that of using Eqs. 4 and 6, respectively, as objective functions with constraints taken from Table 3. The control conditions used for optimization are shown in Table 7. For initialization in step 2, the lower and upper bounds of the control parameters of Table 3 are used to obtain the initial population.

6 Results and discussion

The burr formation optimization problem in milling without considering burr type can now be fully mathematically stated as follows:

$$\begin{aligned} B_h = &-11.3217 + 0.6572x_1 - 2.4616x_2 - 0.0027x_3 + 0.0044x_4 \\ &- 0.044x_1x_2 + 0x_1x_3 - 0x_1x_4 + 0.0133x_2x_3 - 0.0011x_2x_4 \\ &- 0x_3x_4 - 0.0084x_1^2 - 2.4626x_2^2 - 0x_3^2 - 0x_4^2 \end{aligned} \tag{7}$$

$$\begin{aligned} \text{s.t.}\quad & 19 \leq x_1 \leq 55 \\ & 0.5 \leq x_2 \leq 2.0 \\ & 65 \leq x_3 \leq 500 \\ & 320 \leq x_4 \leq 865 \end{aligned} \tag{8}$$

Whereas the burr formation optimization problem in milling considering burr type can now be fully mathematically stated as follows:

$$\begin{aligned} B_h = &-8.8600 + 0.5205x_1 - 3.1938x_2 - 0.0026x_3 + 0.0124x_4 \\ &+3.5647x_5 + 0.0449x_1x_2 + 0.0003x_1x_3 - 0.0002x_1x_4 \\ &-0.1319x_1x_5 + 0.0166x_2x_3 - 0.0006x_2x_4 - 2.4828x_2x_5 \\ &-0.0000x_3x_4 - 0.0100x_3x_5 + 0.0065x_4x_5 \\ &-0.0064x_1^2 - 0.8028x_2^2 - 0.0000x_3^2 - 0.0000x_4^2 + 3.5647x_5^2 \end{aligned} \tag{9}$$

With constraints:

$$\begin{aligned} \text{s.t.}\ & 19 \leq x_1 \leq 55 \\ & 0.5 \leq x_2 \leq 2.0 \\ & 65 \leq x_3 \leq 500 \\ & 320 \leq x_4 \leq 865 \\ & 0 \leq x_5 \leq 1 \end{aligned} \tag{10}$$

DE was used for optimizing the objective functions for burr height without considering burr type and burr height considering burr type. For the experimentation, the optimal cutting parameters and burr size that the DE found are given in Table 8. The approach of using the hybrid GMDH model Eq. 7 or 9, respectively, as objective functions with constraints given in Eq. 8 or 10 for optimizing the burr problem is more straightforward than when ANN is employed in modeling. This type of mathematical formulation makes the hybrid GMDH response models to be more useful to the end-user since the models for the problem being solved are transparent and could be used for future applications. Moreover, the mathematical models are easy to be used as the objective functions by most standard optimization techniques for determining optimal cutting and response conditions. From the investigation, the optimal conditions are found to be simultaneous matching of low feed and drill diameter on one hand with large point angle on the other hand.

7 Experimental validation

In order to provide the experimental evidences on how well and reliable the resulted optimal model can be, the optimal values obtained and listed in Table 8 (in-plane exit angle=55°,

Table 7 DE control parameters used for experimentation

Population size, NP	50
Number of parameter	4 or 5
Mutation probability, F	0.20
Crossover probability, CR	0.60
Number of generations	500

Table 8 Optimal cutting parameters from DE

Parameter	Case 1	Case 2
In-plane exit angle (deg)	55	55
Depth of cut (mm)	2.0	2.0
Feed rate (rev/min)	65	65
Cutting speed (rpm)	585.665	793.701

depth of cut=2 mm, feed rate=65 rev/min, and cutting speed=600 rpm) were used for milling some specimen and the burr levels were measured. It should be noted that the cutting speed of 585.665 rpm does not exist on the milling machine used for experimentation; hence, the value of 600 rpm was used. For three milling replications, the average burr measured were 0.2298, 0.2735, and 0.1109 mm, respectively.

It could be observed from Table 4 in row 3 that the minimum burr height is 0.116 mm occurs for an in-plane exit angle=19°, depth of cut=0.5 mm, feed rate=127 65 rev/min, and cutting speed 600 rpm. The DE optimization obtained a value of 0.1109 mm which is lower than the least measured burr height in Table 4.

Further benchmarking studies were undertaken by comparing the results of DE and SOMA for the optimal cutting parameters for case 1. While the DE optimization approach obtained optimal parameters resulting in burr value of 0.1109 mm, the SOMA optimization approach obtained optimal parameters resulting in burr value of 0.2514 mm, which is inferior. The results of the benchmarking studies are shown in Table 9.

Consequently, an experimental validation has been carried out to justify the fact that the optimal values obtained using DE optimization approach, are indeed reliable. The validation exercise was carried out to evaluate how consistent the optimal model is with respect to the experimental data, and is not for the measurement of the goodness of the optimal model itself. Table 10 shows the ANOVA results when burr types are considered. From the ANOVA results in Table 10, it could be inferred from the sequential sums of squares that the in-plane exit angle is most significant followed by the cutting speed, then depth of cut, and finally feed rate.

8 Conclusions

This paper has presented a new methodology for predictive modeling and optimization for burr size minimization in milling of aluminum using hybrid GMDH network model. Mathematical models were formulated based on the control parameter inputs and predicted outputs using the presented hybrid GMDH network model. These mathematical models were used as objective functions for optimization using DE in order to determine the optimal combination values of in-plane exit angle, depth of cut, feed rate, and cutting speed. The optimal conditions are found to be simultaneous matching of low feed on one hand and fairly high cutting speed on the other hand. A number of important outcomes of the investigation could be summarized as follows:

1. The burr formation optimization problem in milling without considering burr type is represented mathematically so that in the future, such mathematical model could be used to predict burr formation;
2. The burr formation optimization problem in milling considering burr type is represented mathematically so that in the future, such mathematical model could be used to predict burr formation;
3. Including the burr type as a parameter results in a classification scheme in which the burr type becomes the group label and it is therefore possible in the future to classify a machining process into any of these burr types;
4. The modeling approach is applicable to general conditions of the milling process for a variety of materials and cutting conditions.

Table 9 Optimal cutting parameters from DE and SOMA for case 1

	DE	SOMA
In-plane exit angle (deg)	55	55
Depth of cut (mm)	2.0	2.0
Feed rate (rev/min)	65	65
Cutting speed (rpm)	585.665 (600)	320
Minimum burr height (mm)	0.1109	0.2514

Table 10 ANOVA results when burr types are considered

	DF	SS	MS	*F*
Regression	5	43.81	8.76	2.6e+020
Residual error	16	0.00	0.00	
Total	21	43.81		
R^2	1.00000			
Source	Seq SS			
In-plane exit angle (deg)	0.0001			
Depth of cut (mm)	0.5543			
Feed rate (rev/min)	5.9075			
Cutting speed (rpm)	0.0038			

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